GRADE : 11

DATE : 2 / 6 / 2017

SUBJECT : Mathematics

TITLE : Paper 1

SOLUTIONS

EXAMINER : Mr A. Slaughter

TOTAL MARKS : 100

TIME : 2 hour(s)

1.1.	1. $3x(x-4) = 5$	
	$3x^2 - 12x - 5 = 0$ ✓	
	$(\quad)(\quad) = 0$	
	$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-5)}}{2(3)}$ ✓	
	$= \frac{12 \pm \sqrt{204}}{6}$	
	$= 4{,}38$ or $-0{,}38$ ✓ ✓	4
1.1.	2. $3\sqrt{2-x} + 2x = -5$	
	$(3\sqrt{2-x})^2 = (-2x-5)^2$	
	$9(2-x) = 4x^2 + 20x + 25$ ✓	
	$18 - 9x =$ ✓	
	$0 = 4x^2 + 29x + 7$ ✓	
	$= (4x+1)(x+7)$ ✓	
	$\therefore x = \cancel{-\frac{1}{4}}$ or -7	6
	reject ✓ ✓	
1.1.	3. $\frac{3x+1}{7x-1} = \frac{2x-1}{3x+1}$	
	$(3x+1)(3x+1) = (2x-1)(7x-1)$ ✓	
	$9x^2 + 6x + 1 = 14x^2 - 9x + 1$	
	$0 = 5x^2 - 15x$	
	$\div 5$: $0 = x^2 - 3x$ ✓	
	$= x(x-3)$ ✓	
	$\therefore x = 0$ or 3 ✓	4
1.1.	4. $2^x - 5.2^{x-3} = 3\sqrt{2}$	
	$2^x - 5.2^x.2^{-3} = 3.\sqrt{2}$	
	$2^x(1 - 5.2^{-3}) =$	
	$2^x\left(1 - \frac{5}{2^3}\right) =$	
	$2^x\left(1 - \frac{5}{8}\right) =$ ✓	
	✓ $2^x . \frac{3}{8}$ ✓ $= 3.(2^{\frac{1}{2}})$	
	$\times \frac{8}{3}$: $2^x = 2^{\frac{1}{2}}.8$	
	$= 2^{\frac{1}{2}}.2^3$	
	$= 2^{7/2}$	
	$\therefore x = 7/2$ ✓	4

1.1. 5. $2x^{-3} - 3x^{-3/2} = 5$

$k = x^{-3/2}$

$(k)^2 = (x^{-3/2})^2$

$= x^{-3}$

$2k^2 - 3k - 5 = 0$

$(2k-5)(k+1) = 0$

$\therefore k = 5/2$ or $k = -1$ both

$x^{-3/2} = 5/2$ no soln

$x = \left(\frac{5}{2}\right)^{-2/3}$

$= 0{,}54$

5

no mark for $^{-3/2}\sqrt{}$

1.1. 6. $12 > x(6x+1)$

$0 > 6x^2 + x - 12$

$0 > (3x-4)(2x+3)$

$+ \quad 0 \quad (-) \quad 0 \quad +$

$-3/2 \qquad\qquad 4/3$

$\underbrace{-3/2 < x < 4/3}$

4

1.1. 7. $x \geq 2$

1

1.2. $x^2 - xy - y^2 = -1$

$2^x \cdot 32 = 4^y$

$2^x \cdot 2^5 = (2^2)^y$

$2^{x+5} = 2^{2y}$

$x + 5 = 2y$

$x = 2y - 5$

$(2y-5)^2 - (2y-5)y - y^2 = -1$

$4y^2 - 20y + 25 - 2y^2 + 5y - y^2 + 1 = 0$

$y^2 - 15y + 26 = 0$

$(y-13)(y-2) = 0$

$y = 13$ or $y = 2$ both

$x = 2(13) - 5 \qquad x = 2(2) - 5$

$= 21 \qquad = -1$ both

$\therefore x = 21$ and $y = 13$

or

$x = -1$ and $y = 2$

7

1.3. $49x^2 - 42x + 9 = 0$

$\Delta = (-42)^2 - 4(49)(9)$ ✓

$= 0$ ✓

$\therefore$ roots are
- real
- rational
- equal

$\Big\}$ ✓ ans only (no Δ) 0/3

3

1.4. 1. $y = x + \frac{1}{x}$

LCD $= x$ $(\therefore x \neq 0)$

$\times$ thru

$yx = x^2 + 1$

✓ $0 = x^2 - yx + 1$

1

1.4. 2. $\Delta = (-y)^2 - 4(1)(1)$ ✓

$= y^2 - 4$ ✓

2

1.4. 3. For a real graph

$\Delta \geqslant 0$

$y^2 - 4 \geqslant 0$ ✓

$(y-2)(y+2) \geqslant 0$ ✓

$\oplus \quad 0 \quad - \quad 0 \quad \oplus$

$\quad -2 \qquad\quad 2$

$y \leq -2$ or $2 \leq y$

✓ ✓ -1 no "or"

4

1.5. $\dfrac{(2 - 3\sqrt{5})^2}{\sqrt{5}}$

num $= (2 - 3\sqrt{5})(2 - 3\sqrt{5})$

$= 4 - 12\sqrt{5} + 9.5$

$= 49 - 12\sqrt{5}$ ✓

$\dfrac{49 - 12\sqrt{5}}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$ ✓

$= \dfrac{49\sqrt{5} - 12.5}{5}$

$= \dfrac{49\sqrt{5}}{5} - 12$

$= -12 + \frac{49}{5} \cdot \sqrt{5}$

✓ ✓

4

2. $A(-2; -4) \xrightarrow[3 \leftarrow]{5 \uparrow} (-5; 1)$ ✓ ✓

2

3. $f(x) = -2.3^{x-1} + 5$

$y = -2.3^{x-1} + 5$

3.1. • exponential

• yint: $y = -2.3^{0-1} + 5$

$= \frac{13}{3}$ 4,33

• xint: $0 = -2.3^{x-1} + 5$

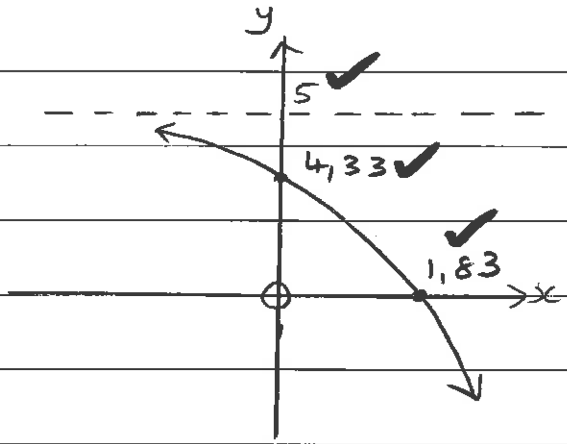
$2.3^{x-1} = 5$

$3^{x-1} = \frac{5}{2}$

$x - 1 = \frac{\log 5/2}{\log 3}$ ✓ log 3

$x = 1,83$

• ha: $y = 5$



5

3.2. $x = -1$ $y = -2.3^{-1-1} + 5$

$= \frac{43}{9}$ ✓

$\therefore (-1; \frac{43}{9})$

$x = 1$ $y = -2.3^{1-1} + 5$

$= 3$ ✓

$\therefore (1; 3)$

av grad

$= \frac{\Delta y}{\Delta x}$

$= \frac{3 - 43/9}{1 - (-1)}$

$= -\frac{8}{9}$ ✓ $-0,89$ 3

4. $f(x) = \frac{3x-8}{x-2}$

$y = \frac{3x-8}{x-2}$

4.1.

$$\begin{array}{r} 3 \\ x-2\,\overline{)\,3x-8} \\ \ominus\ \ \oplus\quad \\ 3x-6 \\ \hline -2 \end{array}$$

✓ method

$\therefore \frac{3x-8}{x-2} = 3 + \frac{-2}{x-2}$ **1**

NO MARKS FOR :

$-\frac{2}{x-2} + 3 = \frac{-2+3(x-2)}{x-2}$

$= \frac{-2+3x-6}{x-2}$

$= \frac{3x-8}{x-2}$

4.2. 1. ha : $y = 3$

va : $x = 2$

$\therefore A(2;3)$ ✓✓ **2**

4.2. 2. yint : $y = \frac{3(0)-8}{0-2}$

$= 4$

$\therefore B(0;4)$ ✓ **1**

4.2. 3. xint : $0 = -\frac{2}{x-2} + 3$

$\frac{2}{x-2} = 3$

LCD = $(x-3)$ $(\therefore x \neq 3)$

× thru

$2 = 3(x-2)$

$\frac{8}{3} = x$ 2,67

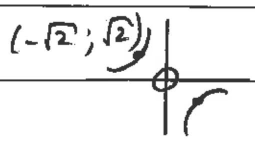
$\therefore C(\frac{8}{3};0)$ ✓✓ **2**

4.3. 1. $y = -(x-2)+3$

$= -x+2+3$

$= -x+5$ ✓✓ **2**

−1 if no $y = \ldots$

4.3. 2. 

$\therefore P(-\sqrt{2}+2;\sqrt{2}+3)$ OK

$= P(0,59;4,41)$ ✓✓ **2**

4.4.	f: $y = -\frac{2}{x-2} + 3$	
	Reflect x axis	
	$\cdot\ x \to x$	
	$\cdot\ y \to -y$	
	$-y = -\frac{2}{x-2} + 3$ ✓	
	✓ $y = \frac{2}{x-2} - 3$	
	OR	
	$-y = \frac{3x-8}{x-2}$ ✓	
	$y = -\frac{3x-8}{x-2}$ ✓	2
4.5.	$x.\ f(x) \geqslant 0$	
	$x.\ y_f\ \ 0+$	
	$x \in [0;2)$ or $[\frac{8}{3};\infty)$	2
	✓ val + not^n ✓ val + not^n	

5.	f. $y = ax^2 + bx + c$	
	g: $y - 2x - 10 = 0$	
	ie $y = 2x + 10$	
5.1.1.	yint: $y = 10$	
	$\therefore C(0;10)$ ✓	1
5.1.2.	xint: $0 = 2x + 10$	
	$-5 = x$	
	$\therefore A(-5;0)$ ✓	1
5.1.3.	$\frac{x_B + (-5)}{2} = -2$	
	$\therefore x_B = 1$	
	3 3 -5 -2 1	
	$\therefore B(1;0)$ ✓	1
5.2	$y = a(x+5)(x-1)$ ✓	
	snb $C(0;10)$	
	$10 = a(5)(-1)$ ✓	
	$-2 = a$ ✓	

$\therefore y = -2(x+5)(x-1)$

$= -2(x^2+4x-5)$

$= -2x^2 - 8x + 10$ | 4

5.3. $x = -2$

$y = -2(-2)^2 - 8(-2) + 10$

$= 18$

$\therefore M(-2; 18)$ | 1

5.4. $f(x).g(x) \leq 0$

$y_f \cdot y_g - 0$

$x = -5$ or $x \in [1; \infty)$ | 2

5.5. 1. HL

method $= y_H - y_L$ $\quad y_{line}$

$= -2x^2 - 8x + 10 - (2x+10)$

$= -2x^2 - 8x + 10 - 2x - 10$

$= -2x^2 - 10x$ | 2

5.5. 2. $x = -\frac{(-10)}{2(-2)}$

$= -\frac{5}{2}$ | 1

5.5. 3. HL_{max}

$= -2(-\frac{5}{2})^2 - 10(-\frac{5}{2})$

$= \frac{25}{2}$ $\quad 12,5$ | 1

5.6. $2k - 5 = 2x^2 + 8x$

$-2x^2 - 8x + 2k - 5 = 0$

$-2x^2 - 8x + \underbrace{2k-5}_{y\,int} = y$

$10 - 18 < y\,int < 0$

$-8 < 2k - 5 < 0$

$-3 < 2k < 5$

$-\frac{3}{2} < k < \frac{5}{2}$ | 3

6.1. $-7 ; -15 ; -23$

$-8 \quad -8$

$D_n = a + (n-1)d$

$= -7 + (n-1)(-8)$

$= -7 - 8n + 8$

$= -8n + 1$

2

6.2. $D_n = -8n + 1$

$-1199 = -8n + 1$

$8n = 1200$

$n = 150$

$\therefore T_{150}$ and T_{151}

3

6.3. $d_2 = 2a \qquad d_1 = 3a + b$

$-8 = 2a \qquad -7 = 3(-4) + b$

$-4 = a \qquad 5 = b$

$T_{38} = -5576$

$n = 38$

$a(38)^2 + b(38) + c = -5576$

$-4(38)^2 + 5(38) + c = -5576$

$\therefore c = 10$

$\therefore T_n = -4n^2 + 5n + 10$

5